

Naturalna dedukcja

Zadanie 609.4)

$$\begin{array}{c}
 \frac{\frac{\frac{}{\Gamma, p \vdash p} \text{(Ax)} \quad \frac{\frac{}{\Gamma, p \vdash p \rightarrow q \vee r} \text{(Ax)}}{\Gamma, p \vdash q \vee r} \text{(W}\rightarrow\text{)}}{\Gamma, q \vdash q} \text{(Ax)} \quad \frac{\frac{\frac{}{\Gamma, r \vdash r} \text{(Ax)}}{\Gamma, r \vdash p \rightarrow q \vee r} \text{(W}\vee\text{)}}{\Gamma, r \vdash \neg(p \rightarrow q \vee r)} \text{(E}\neg\text{)}}{\Gamma, r \vdash \perp} \text{(E}\perp\text{)}}{\Gamma, r \vdash q} \text{(E}\vee\text{)}} \\
 \frac{\frac{\frac{}{\Gamma, p \vdash q} \text{(Ax)}}{\Gamma \vdash p \rightarrow q} \text{(W}\rightarrow\text{)}}{\Gamma \vdash (p \rightarrow q) \vee r} \text{(W}\vee\text{)}} \quad \frac{\frac{\frac{}{\Gamma \vdash \neg(p \rightarrow q \vee r)} \text{(Ax)}}{\Gamma := (p \rightarrow q \vee r), \neg((p \rightarrow q) \vee r) \vdash \perp} \text{(E}\neg\neg\text{)}}{\frac{(p \rightarrow q \vee r) \vdash (p \rightarrow q) \vee r}{\vdash (p \rightarrow q \vee r) \rightarrow (p \rightarrow q) \vee r} \text{(W}\rightarrow\text{)}}}{\Gamma \vdash \neg(p \rightarrow q \vee r)} \text{(E}\neg\text{)}}
 \end{array}$$